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STUDY OF THE INFLUENCE OF VARIABILITY IN THE DISTANCE BETWEEN CRACKS ON THE TYPE OF STRENGTH DISTRIBUTION OF STRUCTURAL ELEMENTS

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Abstract. *The considered theories and concepts of strength are based on a model of a body either as a homogeneous structureless medium, or as a material that has a structure, but is homogeneous throughout its entire volume. Rocks are certainly not such bodies. They are composed of mineral grains of different properties, contain macrodefects in the form of pores and various inclusions, as well as objects of different states of aggregation (gases, liquids). Under these conditions, deterministic theories of strength are clearly untenable.*

Keywords: Griffiths crack theory, statistical concept, critical stress, empirical coefficient, material constant, mountain shear

Mathematics Subject Classification (2020): 74S70, 65E05

1. Introduction

In particular, the use of the classical Griffiths theory of cracks is complicated by the following circumstance. Since the rock is an aggregate of mineral grains, a microcrack developing inside the grain inevitably reaches its boundary and, consequently, the radius r of the crack mouth increases abruptly. Therefore, for a crack to transition to another grain and its further development, a voltage greater than that follows from Griffiths' theory is required. Thus, there is a certain "barrier" stress at which only the development of a crack in real rock is possible. In addition, the development of cracks in rock occurs predominantly along the contact of mineral grains, i.e., along the cementing material, often of a clayey composition. For such a material, the theory of brittle fracture is not applicable.

The destruction of rock (from the standpoint of any strength theory) is determined by the stresses acting in it. But due to the heterogeneous structure of the rocks, local centers

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of stress concentration are randomly distributed throughout its volume. Therefore, the strength and destruction of rocks must be considered from a statistical point of view. This approach is also justified for most other materials used by humans.

2. Formulation of the Problem

Mathematically, a crack is an interface along which a displacement vector undergoes a discontinuity. A trace of this surface in the plane

$$Ax + By + C = 0,$$

which is the general equation of the straight line L, where A, B, C are constant numbers, coordinates of the normal vector of the straight line L, and their special values distinguish this line from many others.

Dividing both of its parts by the normalizing factor $\sqrt{A^2 + B^2}$, we obtain the normal equation of the line

$$x \cos\alpha + y \sin\alpha - l = 0,$$

where $\cos\alpha = \frac{A}{\sqrt{A^2+B^2}}$, $\sin\alpha = \frac{B}{\sqrt{A^2+B^2}}$—are the direction cosines of the line, l is the distance from the line to the origin. If a line is drawn through the origin parallel to line L, to l will be equal to the distance between the lines.

So, the equation

$$x \cos\alpha + y \sin\alpha - nl = 0, \quad n = 0, 1, 2, ...,$$

where l is the distance between cracks, determines the system of cracks on the plane (Fig. 1).

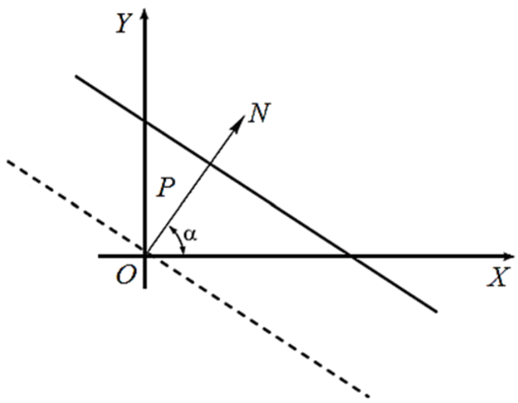

Fig. 1. To determine the distance between cracks on a plane

M.V. Gzovsky [9] proposed using modern knowledge about the physical conditions of their occurrence when studying tectonic ruptures. In particular, the patterns of occurrence of elementary surfaces of mechanical destruction must be considered taking into account the fact that the destruction process develops over a long period of time and continuously together with elastic and plastic deformation, in parallel with the process of healing of ruptures. An important consequence of the formation of a tectonic rupture is a change in the primary stress state near it. It follows that the location of each crack of a certain order depends on the position of the cracks adjacent to it.

Thus, the distance between cracks is the main parameter of fracturing. This value was formed under the influence of a large number of random events and is itself random. But as a result of the manifestation of mass random events, it is subject to a certain law of probability distribution.

S.A. Batugin collected a lot of statistical information about the dimensions of the distances between the cracks [10]. For the construction of empirical distributions, the results of measurements in homogeneous distribution areas in some mining enterprises were selected. It is clear from (Fig. 2) that the distribution of the random quantity is asymmetric. The author notes that several statistical hypotheses are not rejected at the 5% significance level according to Pearson's criterion (typical for small samples). However, based on the physical nature of this random variable, Rayleigh's law should be considered as a probabilistic model for the crack spacing distribution.

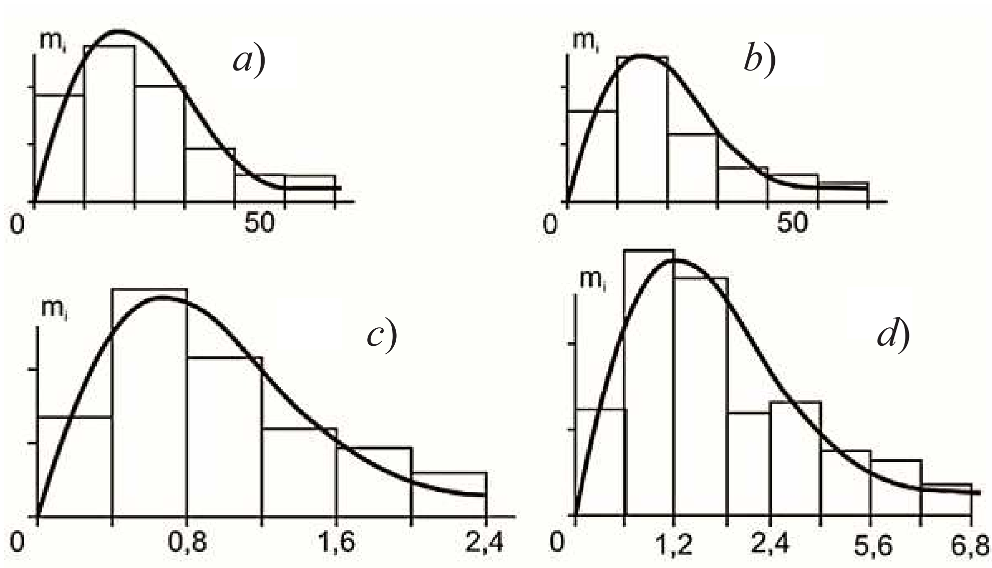

Fig. 2. Distribution of distances between cracks in rocks

Indeed, Rayleigh's law is derived as a statistical model of the distance between two points of the plane. If X and Y are normal independent random variables with zero mathematical expectations and equal mean squared deviations σ, then the

$$l = \sqrt{X^2 + Y^2},$$

distributed according to Rayleigh's law with density [2]:

$$f(x,\sigma) = \begin{cases} \frac{x}{\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right), & x \geq 0, \quad \sigma > 0, \\ 0, & x < 0. \end{cases}$$

This distribution contains only the scale parameter σ, with which the mathematical expectation m is associated with a very simple dependence

$$\sigma = 0,52m. \tag{1}$$

According to S.A. Batugin, the types of rocks he examined (sandstones, shales) are characterized by a relative variation in the distances between cracks, amounting to 50-55%. It is indicated that relation (1) is satisfactorily observed for all examined crack systems.

Following the chosen statistical model, we determine the probability that the distance between the cracks of a certain system will be no less than a certain critical value l^*:

$$P\left(l \geq l^*\right) = 1 - \int_0^{p^*} \frac{x}{\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx = \exp\left(-\frac{l^{*^2}}{2\sigma^2}\right).$$

Resolving this equation for l^*, we obtain:

$$l^{*2} = -2\sigma^2 \ln p. \tag{2}$$

If we consider expression (1) in (2), we have the following expression

$$l^{*2} = \sigma\sqrt{-2\ln p} = 0.52m\sqrt{-2\ln p}.$$

For example, with probability $P = 0.65$ all values of l^* will be no less than

It can be shown that the entire initial moment of the "corrected" order and the k arrangement is related by the following expression [4]:

$$m'_k = K_k m_k,$$

here

$$K_k = \frac{\nu + f^k(\alpha)}{\nu + 1} = \frac{\frac{l_m}{l_0} + f^k(\alpha)}{\frac{l_m}{l_0} + 1}. \tag{3}$$

Taking into account the relations obtained, when determining the effect coefficient of cracks in formula (3), we assume that

$$\frac{l_m}{l_0} = \frac{m}{l_0} 0,52\sqrt{-2\ln p}.$$

In the special case, if we consider the strength of damaged elements equal to zero, the effect coefficient of cracks is the same for all initial moments:

$$K_1 = K_2 = K_3 = ...K = K = \frac{\frac{l_m}{l_0}}{\frac{l_m}{l_0} + 1} = \frac{l_m}{l_m + l_0}. \tag{4}$$

In expression (4), the value of K varies from 0.5 ($l_m = l_0-$highly cracked medium) to 1.0 ($l_m \to \infty-$uncracked medium).

$$\frac{l_m}{l_0} = \frac{m}{l_0} 0.52\sqrt{-2\ln p}.$$

The central moments of the second, third and fourth orders are associated with normalized indicators of asymmetry β_1 and kurtosis β_2:

$$\beta_1 = \frac{\mu_3}{\mu_2^{\frac{3}{2}}}, \quad \beta_2 = \frac{\mu_4}{\mu_2^2}. \tag{5}$$

The empirical center $\beta_2 = \frac{\mu_4}{\mu_2^2}$. moments of distribution of statistical data is determined:

Distributions for empirical data can be made using a Pearson diagram, which presents theoretical distributions depending on their characteristic skewness and kurtosis values. The latter are determined by the central moments of the third and fourth orders (formulas (5)). In turn, the central moments can be expressed through the initial ones:

$$\begin{aligned} \mu_2 &= m_2 - m_1^2, \\ \mu_3 &= m_3 - 3m_2 m_1 + 2m_1^3, \\ \mu_4 &= m_4 - 4m_3 m_1 + 6m_2 m_1^2 - 3m_1^4. \end{aligned} \tag{6}$$

For a normal distribution, all initial moments of odd orders are equal to zero. From this we obtain the well-known relations [3]:

$$\beta_1^2 = \frac{\mu_3^2}{\mu_2^3} = 0, \tag{7}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = 3. \tag{8}$$

Let us determine from these conditions how the moments of a symmetrical (normal) distribution should relate to each other. From the second equation (6) and condition (7) we obtain that

$$\mu_3 = m_3 - 3m_2 m_1 + 2m_1^3 = 0,$$

or

$$\frac{m_3}{m_1^3} = 3\frac{m_2}{m_1^2} - 2.$$

Then expressions (6) for the central moments will take the form [8]:

$$\begin{aligned} \mu'_2 &= K_2 m_2 - K_1^2 m_1^2, \\ \mu'_3 &= K_3 m_3 - 3K_1 K_2 m_2 m_1 + 2K_1^3 m_1^3, \\ \mu'_4 &= K_4 m_4 - 4K_3 K_1 m_3 m_1 + 6K_2 K_1^2 m_2 m_1^2 - 3K_1^4 m_1^4. \end{aligned}$$

As we have seen, the presence of elements distorted by macrodefects changes all moments of the distribution, including those that determine asymmetry and kurtosis. Considering relations (6) and (8), the asymmetry indicator can be expressed in relative values [5]:

$$\beta_1^2 = \frac{(A_3 - 3A_2 + 2)^2}{(A_2 - 1)^3}.$$

For the "corrected" series, which includes elements with macrodefects, we obtain

$$A'_2 = \frac{K_2 m_2}{K_1^2 m_1^2} = \frac{K_2}{K_1^2} A_2, \quad A'_3 = \frac{K_3 m_2}{K_1^3 m_1^2} = \frac{K_3}{K_1^3} A_3, \quad A'_4 = \frac{K_2 m_2}{K_1^2 m_1^2} = \frac{K_4}{K_1^4} A_4.$$

The coefficient of variation of conventional sampling (without taking into account disturbed samples) is determined by the formula:

$$\eta = \frac{\sqrt{D}}{m_1}.$$

Let us introduce the notation

$$A_k = \frac{m_k}{m_1^k},$$

Then

$$\eta^2 = \frac{m_2 - m_1^2}{m_1^2} = A_2 - 1,$$

were

$$A_2 = \eta^2 + 1.$$

Then the asymmetry indicator will take the form:

$$\beta_1^{2'} = \frac{\left(\frac{K_3}{K_1^3} A_3 - 3\frac{K_2}{K_1^2} A_2 + 2\right)^2}{\left(\frac{K_2}{K_1^2} A_2 - 1\right)^3}. \tag{9}$$

Similarly, we obtain an expression for the kurtosis indicator:

$$\beta'_2 = \frac{\left(\frac{K_4}{K_1^4} A_4 - 4\frac{K_3}{K_1^3} A_3 + 6\frac{K_2}{K_1^2} A_2 - 3\right)^2}{\left(\frac{K_2}{K_1^2} A_2 - 1\right)^3}. \tag{10}$$

$\frac{m}{l_0}$	ν	$\beta_1^{2'}$	β'_2
1.0	0.5	1.70	5.71
1.5	0.6	1.08	4.98
2.0	0.66	0.87	4.63
2.5	0.71	0.98	4.01
3.0	0.75	0.68	3.58
10	0.9	0.17	3.22

Table. *Values of asymmetry and kurtosis indicators of the "corrected" variation series, taking into account the random distribution of distances between cracks*

Table shows the values of the asymmetry and kurtosis indicators $\beta_1^{2'}$, β' (formulas (9) and (10)), obtained for various values of $\frac{m}{l_0}$ under the assumption that the distances between cracks are not less than the value $l^* = m - \sigma$. Calculations were performed with $f(\alpha) = 0.3$ and $A_2 = 1.2$.

When the average distance between cracks is smaller than the sample size, the points on the β_1^2, β_2 plane are close to the gamma distribution. But as the ratio $\frac{m}{l_0}$ approaches unity, the distribution tends to be logarithmically normal, which most closely corresponds to the physical essence of the random variable under study-the compressive strength of structural elements.

Let us return to the question of how the type of probability distribution function of the strength of structural elements determines the value of the strength of the rock mass as a whole according to the following equation.

Following statistical theories of strength [1], a rock mass can be represented as a certain aggregate consisting of structural elements. Due to the heterogeneity of the rock environment, the strength of structural elements is a random variable and obeys one or another probability distribution law with the distribution density $f(R)$ (Fig. 3).

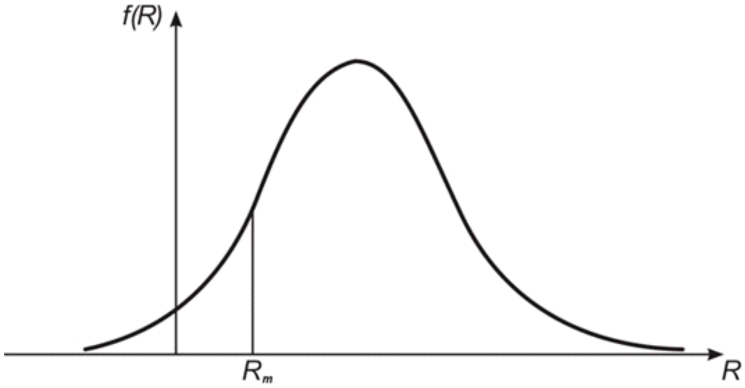

Fig. 3. Hypothetical distribution of strength of structural elements of a rock mass

The difference between the strength of the massif (aggregate) R_m (Fig. 3) and the mathematical expectation of the strength of structural elements $M(R)$ is estimated by the coefficient of structural weakening equal to

$$k_c = \frac{R_m}{M(R)}. \tag{11}$$

The strength of the array should be assessed by such a value R_m that the strength of its structural elements, including laboratory samples, with a given reliability, is not less than this value. The probability of such an event is determined by the expression

$$P\left(R \geq R_m\right), \tag{12}$$

where $F(R) = \int\limits_{-\infty}^{R} f(x)dx$ is the integral distribution function of the value R.

Let us resolve this inequality with respect to the value of R_m:

$$R_m = \arg F(1-P),$$

where $\arg F(1-P)$ is the argument of the function $F(R)$ with its value equal to $1-P$.

Then the structural weakening coefficient is determined by the expression [6]:

$$k_c = \frac{\arg F(1-P)}{M(R)},$$

the specific form of which depends on the choice of the probability distribution function $F(R)$ of the random variable R the strength of structural elements.

As a rule, the choice of the distribution law is carried out based on the physical essence of the random variable and the analysis of statistical information. Most often, especially when the volume of such information is small, researchers choose the normal distribution law as a probabilistic model of the quantitative characteristic under study. In this case, they are guided by the central limit theorem and the law of large numbers, from which the conclusion follows: if the variation of a random variable occurs under the influence of a large number of independent factors, and the influence of each of them is insignificant compared to the cumulative influence of other factors, then the distribution of the random variable obeys the normal law. Since the conditions that define a normal distribution occur frequently, the latter has become widespread. The advantage of the normal distribution is that its parameters have a clear physical meaning.

Indeed, the distribution density of a random variable subject to the Gaussian law has the form:

$$f(R) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{(R-a)^2}{2\sigma}},$$

where a is the mathematical expectation of the value R; $\sigma$is its standard deviation.

Let us obtain the value of the structural weakening coefficient under the assumption that the strength of the structural elements of the massif is distributed according to the normal law. In this case, inequality (12) takes the form

$$P\left(R \geq R_m\right) = 1 - F_0\left(\frac{R_m - a}{\sigma}\right), \tag{13}$$

were

$$F_0(t) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{t} e^{\frac{t^2}{2}} dt, \tag{14}$$

normalized normal distribution function.

Let us resolve equation (13) with respect to the value of R_m:

$$F_0 = \left(\frac{R_m - a}{\sigma}\right) = 1 - P,$$

$$\frac{R_m - a}{\sigma} = \arg F_0(1-P),$$

where $t = \arg F_0(1-P)$ is the argument of function (14) with its value $F_0(t)$ equal to $1-P$.

Next we get:

$$R_m = \sigma \cdot \arg F_0(1-P) + a.$$

Considering that $M(R) = a$, dividing both parts of the resulting expression by the value a, we obtain:

$$k_c = \frac{\sigma}{a} \cdot \arg F_0(1-P) + 1.$$

Here $\frac{\sigma}{a} = \eta$ is the relative variation in the strength of structural elements. The final expression for the structural weakening coefficient takes the form [7]:

$$k_c = \eta \cdot \arg F_0(1-P) + 1. \tag{15}$$

So, we have obtained the coefficient of structural weakening as a value that depends, firstly, on the relative variation η, which essentially characterizes the degree of heterogeneity of the medium; secondly, on the probability P, which characterizes the level of significance of the object.

Let us determine, for example, the calculated value of the uniaxial compressive strength of siltstone if, according to test data, the average strength of laboratory samples $\overline{R_c}$ is $40MPa$, the variation in values is 30% ($\eta = 0,3$).

From equality (11) it follows that;

$$R_{rajot} = R_m = \overline{R_c} \cdot k_c.$$

Let's set the probability $P = 0,95$. Let us determine the value of the argument t of the normalized normal function $F_0(t)$ at its value equal to $1 - 0,095 = 0,05$.

The value of the integral function $F_0(t) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{t} e^{\frac{t^2}{2}} dt$, equal to, $F_0(t) = 0,05$ corresponds to the value of the argument $t = -1,64$, that is, $\arg F_0(0,05) = -1,64$. Then the structural weakening coefficient is equal to:

$$k_c = 0,3(-1,64) + 1 = 0,508.$$

Thus, the calculated strength value is $R_{rajot} = 0,508 \cdot 40 = 21MPa$.

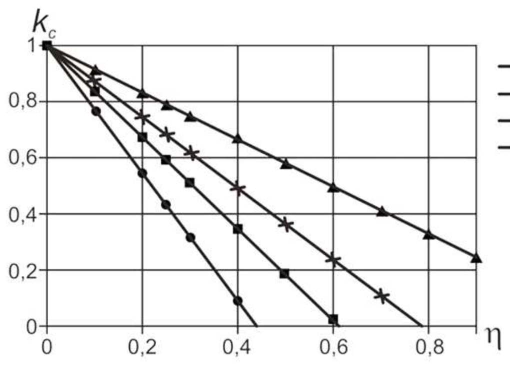

Fig. 4. Dependence of the structural weakening coefficient on the variation in the strength of structural elements under the assumption of a normal law of distribution of their strength

Analyzing the graph of dependence (15) (Fig. 4), we note that when $\eta > 0,4$, the structural weakening coefficient can take negative values, which, naturally, contradicts the physical essence of this value. Obviously, this is a drawback of the probabilistic model. Indeed, integration of the normal distribution density automatically assumes the presence of negative values of R within the limits $-\infty < R < 0$.

It is the quantitative assessment in the form of the structural weakening coefficient that shows the disadvantage of the normal distribution: the uniaxial compressive strength cannot have negative values. The original probabilistic model, attractive for its simplicity, is inadequate for the object under consideration and requires replacement with a more advanced one. Such a more universal probabilistic model is the normal truncated distribution law [11].

The distribution density of the random variable x for the truncated normal law has the form:

$$f(x) = \begin{cases} 0, \quad -\infty < x < x_1, \\ A\left[(2\pi)^{\frac{1}{2}}\sigma\right]^{-1} \exp\left[-\left(x - x_0\right)^2\left(2\sigma^2\right)^{-1}\right], \quad x_1 < x < x_2, \\ 0, \quad x_2 < x < \infty, \end{cases} \tag{16}$$

where x_0 are the first initial and second central moments of the statistical distribution, respectively.

Parameter A in equation (16) is determined from the condition

$$A\frac{1}{\sqrt{2\pi}}\int_{x_1}^{x_2} \exp\left(-\frac{u^2}{2}\right) du = 1,$$

where $u = \frac{(x-x_0)^2}{2\sigma^2}$.

The average strength value and dispersion are found from the expressions

$$M(x) = x_0 + B\sigma, \tag{17}$$

$$D = \sigma^2\left\{1 - B^2 - A\left[\begin{matrix}(x_2 - x_0)\,\sigma^{-1}(2\pi)^{-\frac{1}{2}}\exp\left[-\frac{(x_2-x_0)^2}{2\sigma^2}\right] - \\ -\,(x_1 - x_0)\,\sigma^{-1}(2\pi)^{-\frac{1}{2}}\exp\left[-\frac{(x_1-x_0)^2}{2\sigma^2}\right]\end{matrix}\right]\right\}.$$

Let's denote

$$(2\pi)^{-\frac{1}{2}}\int_0^{\frac{(x_i-x_0)}{\sigma}} \exp\left(-\frac{u^2}{2}\right) du = \varPhi\left[\frac{(x_1 - x_0)}{\sigma}\right],$$

$$\exp\left[-\left(x_i - x_0\right)^2\left(2\sigma^2\right)^{-1}\right] = (2\pi)^{\frac{1}{2}} f\left[\left(x_i - x_0\right)\sigma^{-1}\right]. \tag{18}$$

The quantities A, B, D are determined by the expressions

$$A = \frac{1}{\left[\varPhi\left(\frac{x_2-x_0}{\sigma}\right) - \varPhi\left(\frac{x_1-x_0}{\sigma}\right)\right]},$$

$$B = \frac{f\left(\frac{x_1-x_0}{\sigma}\right) - f\left(\frac{x_2-x_0}{\sigma}\right)}{\varPhi\left(\frac{x_2-x_0}{\sigma}\right) - \varPhi\left(\frac{x_1-x_0}{\sigma}\right)}, \tag{19}$$

$$D = \sigma^2 \left\{ 1 - B^2 - A \left[\frac{x_1 - x_0}{\sigma} f\left(\frac{x_2 - x_0}{\sigma}\right) - \left(\frac{x_2 - x_0}{\sigma}\right) f\left(\frac{x_1 - x_0}{\sigma}\right) \right] \right\}.$$

Let us solve the problem of estimating the strength of a rock mass for a truncated normal distribution law. The strength of the array, as in the previous case, is estimated by the value x with such reliability that during calculations it does not take values less than x_m with probability$p$. The probability that the random variable x will not be lower than the value x_m is equal to:

$$p\,(x_m < x < x_2) = 1 - A(2\pi)^{\frac{1}{2}} \int_{\frac{(x_1 - x_0)}{\sigma}}^{\frac{(x_m - x_0)}{\sigma}} \exp\left(-\frac{u^2}{2}\right) du.$$

Taking into account notations (18) and (19) we obtain

$$p = 1 - \frac{\left[\Phi\left(\frac{x_m - x_0}{\sigma}\right) - \Phi\left(\frac{x_1 - x_0}{\sigma}\right)\right]}{\left[\Phi\left(\frac{x_2 - x_0}{\sigma}\right) - \Phi\left(\frac{x_1 - x_0}{\sigma}\right)\right]}.$$

Let's solve the last equality with respect to x_m - the main characteristic of the strength of the array:

$$x_m = x_0 + \sigma \arg \Phi \left[(1-p)\Phi\left(\frac{x_2 - x_0}{\sigma}\right) + p\Phi\left(\frac{x_1 - x_0}{\sigma}\right) \right]. \qquad (20)$$

The resulting formula for the strength of the rock mass must be determined relative to the statistica characteristics for the truncated normal distribution law, i.e., in formula (20), instead of x_0, it is necessary to take the value $M(x)$ from (17), and instead of σ, respectively, $\sqrt{D}$ from (19). We get

$$x_m = M(x) + \sqrt{D} \arg \Phi \left[(1-p)\Phi\left(\frac{x_2 - x_0}{\sqrt{D}}\right) + p\Phi\left(\frac{x_1 - x_0}{\sqrt{D}}\right) \right].$$

Having divided all terms of the resulting expression by the mathematical expectation $M(x)$, we find a formula for determining the coefficient of structural weakening:

$$k_c = 1 + \eta \arg \Phi [1-p] \Phi\left(\frac{x_2 - x_0}{\sqrt{D}}\right) + p\Phi\left(\frac{x_1 - x_0}{\sqrt{D}}\right). \qquad (21)$$

Thus, formulas have been obtained to determine the calculated strength of the rock mass and the structural weakening coefficient, showing how much, it is necessary to reduce the strength of the rock, found when testing a sample of samples as the mathematical expectation of the truncated normal law, in order to have a calculated strength value. The level of reliability of the obtained estimates is determined by specifying the probability$p$, which depends on the technical or production significance of the designed object.

Figure 5 shows graphs showing how the error that occurs when using the normal distribution law instead of the truncated normal law, which more adequately describes the real rock mass, depends. It follows from the graphs that, depending on the level of reliability, with a coefficient of variation not exceeding 0.2-0.3, the error is 10-13% and under these conditions the normal distribution law and the simpler dependencies arising from it can be applied. At a higher level of rock mass heterogeneity, the error becomes significant and dependencies (20), (21) obtained on the basis of a truncated normal distribution law should be used.

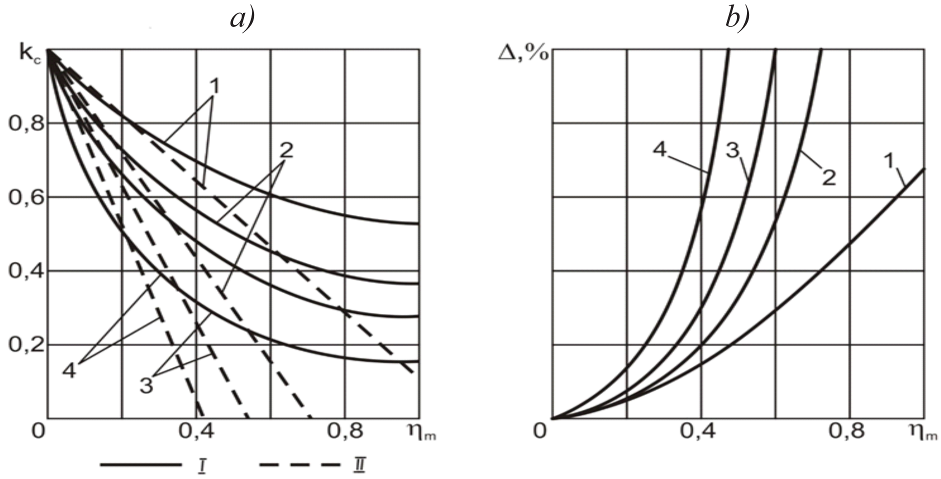

Fig. 5. Dependence of the structural weakening coefficient (a) and the relative calculation error (b) on the relative variation of strength and reliability level: I – truncated normal distribution law; II – normal distribution law; 1,2,3,4 –p=0.8; 0.9; 0.95; 0.99 respectively

3. Conclusions

Thus, the type of distribution of the random variable the strength limit of the structural elements of the rock mass depends on the average distance between the natural cracks of the prevailing system, the spread of the values of these distances and the angle of inclination of the cracks to the loading axis. This is quantitatively reflected in the indicators of asymmetry and kurtosis of the statistical distribution, which serve as a guide for choosing a statistical model of the quantitative characteristic under study. It should be noted that in the general case, the shape of the distribution is not uniquely determined by the indicators of asymmetry and kurtosis. Therefore, hypotheses about the distribution law of random variables, in particular the mechanical characteristics of rocks, should be put forward not only by analyzing their distribution moments and the form of empirical frequency histograms, but also based on the physical essence of these quantities.

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